The Effect of Distortion on Mass Transfer to Spheroidal Drops

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Mass transfer correlations for isolated drops can be used to advantage in the design of liquid extraction equipment (*1*), but these are not available for all of the conditions met in practice. We are concerned here with the effects of droplet distortion resulting from inertial forces. More specifically we consider droplet behavior at low Reynolds and Weber numbers in systems free from surfactants, when the departures from sphericity are small, and we shall use drop shapes and velocity profiles calculated by Taylor and Acrivos (*6*). We shall moreover consider only mass transfer in the continuous phase. The description of this situation gives useful insight into the effect of distortion and suggests the desirability of more extensive analysis when velocity profiles are available for more pronounced distortions.

DROPLET SHAPES AND VELOCITY PROFILES

It has been shown experimentally (*7*) that fluid drops or bubbles moving under the action of gravity through a quiescent second fluid, depart increasingly from a spherical shape as the Reynolds number of the relative motion increases. At low Reynolds numbers, where the distortion is small, the drop shape closely approximates an oblate ellipsoid of revolution; at higher Reynolds number the shape approaches a spherical cap. Taylor and Acrivos (*6*) showed that this distortion results from inertial forces and, for sufficiently slow flow, is dependent only on the Weber number. For small values of Weber number they obtain explicit descriptions of both drop shapes and velocity profiles. The surface of the drop is described by

$$r = R\left[1 - \lambda N_{We} P_2(u)\right] \tag{1}$$

in which

$$\lambda = \frac{1}{4(\kappa+1)^3}\left\{\left(\frac{81}{80}\kappa^3 + \frac{57}{20}\kappa^2 + \frac{103}{40} + \frac{3}{4}\right) - \frac{\gamma-1}{12}(\kappa+1)\right\} \tag{2}$$

P_n is a Legendre polynomial of degree n, and u is used as abbreviation for $\cos\theta$.

The dimensionless stream function for the continuous phase is given by

$$\psi = -\frac{Q_1(u)}{2}\left[\frac{2r^2}{R^2} - \frac{3\kappa+2}{\kappa+1}\frac{r}{R} + \frac{\kappa}{\kappa+1}\frac{R}{r}\right] + \frac{\lambda N_{We}}{10(\kappa+1)^2}\left[(3\kappa^2-\kappa+8)\frac{r}{R} - 3(\kappa^2-\kappa+2)\frac{R}{r}\right]Q_1(u) - \frac{6\lambda N_{We}}{5(\kappa+1)}\left[\left(\frac{3\kappa}{2}+\frac{9}{7}\right) - \left(\frac{3\kappa}{2}+\frac{2}{7}\right)\frac{R^3}{r^3}\right]Q_3(u) \tag{3}$$

where

$$Q_n(u) = \int_{-1}^{u} P_n(x)\, dx \tag{4}$$

and the velocity components are related to the stream function by the following equations:

$$v_\theta = \frac{v_\infty R^2}{r(1-u^2)^{1/2}}\frac{\partial\psi}{\partial r} \qquad v_r = \frac{v_\infty R^2}{r^2}\frac{\partial\psi}{\partial u} \tag{5}$$

The tangential velocity component can be shown to be

$$v_t = v_\theta - v_r(1-u^2)\frac{d}{du}\left[\lambda N_{We} P_2(u)\right] \tag{6}$$

The tangential velocity at the surface of the drop is therefore

$$v_{to} = \frac{v_\infty}{(1-u^2)^{1/2}}\left\{-\frac{Q_1(u)}{\kappa+1} + \lambda N_{We}\left[\frac{Q_1(u)P_2(u)}{\kappa+1}(3\kappa+1) + \frac{Q_1(u)}{5(\kappa+1)^2}(3\kappa^2-2\kappa+7) - \frac{6Q_3(u)}{35(\kappa+1)}(21\kappa-3)\right]\right\} \tag{7}$$

Terms of the order $(N_{We})^2$ and higher are neglected. The reference velocity, v_∞, is that calculated for creeping flow of a circulating spherical drop with the same volume, and R is the radius of such an equivalent sphere.

Mass transfer correlations can now be obtained by integration of the continuity equation using the above velocity profiles. .

MASS TRANSFER BEHAVIOR: SURFACE STRETCH MODIFICATION OF THE PENETRATION THEORY

For liquid drops of low viscosity suspended in another liquid medium, or for gas bubbles, the Peclet number is normally large relative to unity. Under these circumstances significant concentration changes on the outer side of the interface are confined to very thin boundary layers. It is thus permissible to use the surface-stretch model of Angelo, Lightfoot, and Howard (*1*) even though *surface* shear is appreciable. Relative importance of surface shear and velocity have been discussed by Lochiel and Calderbank (*3*), and Byers and King (*2*).

The coordinate system chosen for the present analysis is the same as the system chosen by Stewart (*4*). Locally orthogonal curvilinear coordinates x, y, z are chosen such that y is the distance from the surface along its local normal, x is the distance along the surface following the adjacent stream line and z is distance along the surface perpendicular to the adjacent streamlines. In the interfacial region where the diffusional boundary layer thickness is assumed to be small compared with local radii of curvature, the three coordinates are mutually orthogonal. The average Nusselt number can be obtained following the derivation given by Stewart, the only difference being that the surface is mobile and that the velocity gradient is neglected. The average Nusselt number for the continuous phase in the case of mass transfer from a surface of revolution is

$$N_{Nu} = \frac{2D}{(\pi \mathfrak{D}_{AC})^{1/2}} \frac{\left[\int_0^x v_{to}\, h_x\, h_z^2\, dx \right]^{1/2}}{\int_0^x h_x\, h_z\, dx} \tag{8}$$

where h_x and h_z are scale factors in x and z directions, and D is the characteristic length.

For the oblate spheroids under consideration, the scale factors in spherical coordinates are (3)

$$h_x = \frac{[1 + 2\lambda N_{We}]R}{[1 + 3\lambda N_{We}(1-u^2)]^{3/2}} \tag{9}$$

$$h_z = \frac{(1 + 2\lambda N_{We})(1-u^2)^{1/2}R}{[1 + 3\lambda N_{We}(1-u^2)]^{1/2}} \tag{10}$$

and

$$dx = d\theta = -\frac{du}{(1-u^2)^{1/2}} \tag{11}$$

The average mass transfer Nusselt number can now be computed in a straightforward but lengthy process. The final expression is

$$N_{Nu} = \left(\frac{4}{3\pi}\right)^{1/2} \left(\frac{Dv_\infty}{\mathfrak{D}_{AC}}\right)^{1/2} \left(\frac{1}{\kappa+1}\right)^{1/2} \left[1 + \lambda N_{We} \frac{7\kappa - 17}{20(\kappa+1)}\right] \tag{12}$$

It is interesting to note that this expression predicts an increase of Nusselt number in the case of viscous drops, and a decrease in the case of small gas bubbles ($\kappa \to 0$, $\gamma \to 0$). The correction factor expressed as the fractional difference in Nusselt number for the ellipsoid and a sphere of the same volume is

$$\frac{(N_{Nu})_{\text{oblate spheroid}} - (N_{Nu})_{\text{sphere}}}{(N_{Nu})_{\text{sphere}}} = \lambda N_{We} \frac{7\kappa - 17}{20(\kappa+1)} \doteq -\frac{17}{20}\lambda N_{We}$$

for small gas bubbles. Lochiel and Calderbank (3) have noted that the increase in amount of mass transfer is associated primarily with increase of mass transfer area in the case of spheroids. However for the present analysis the calculated increase in mass transfer area is small. The ratio of area of an oblate spheroid to a sphere having the same volume is

$$\frac{(A)_{\text{oblate spheroid}}}{(A)_{\text{sphere}}} = 1 + \frac{1}{4}\lambda N_{We} \tag{13}$$

Compare this ratio with the ratio of mass transfer coefficient or ratio of Nusselt number:

$$\frac{(N_{Nu})_{\text{oblate spheroid}}}{(N_{Nu})_{\text{sphere}}} = 1 + \lambda N_{We} \frac{7\kappa - 17}{20(\kappa+1)} \tag{14}$$

It is seen that the changes in amount of mass transfer is affected more by the change in velocity profile than by the change in mass transfer area and the overall effect of flattening in this case tends to decrease mass transfer.

Drop phase behavior is complicated by the limited solute capacity of the drop and can differ largely from that of the continuous phase. When fractional extraction is small, the surface stretch modified penetration model is valid for the drop phase too and the phase resistances are additive. The case of spheres has recently been investigated by Taunton and Lightfoot (5), and Ruckenstein (8). Extension of their findings to the case of distorted droplets is now under consideration.

RANGE OF VALIDITY OF THE MODEL

The flow model used in the above development is limited to low N_{Re} and N_{We}. There is as yet no way to express this limitation quantitatively, but the comparison of experimentally observed eccentricities with the Taylor-Acrivos predictions made by Wellek, Agrawal, and Skelland (7) provides a qualitative test. It appears from this comparison that the prediction methods are useful for Reynolds number up to about 20 and for Weber number less than unity. At the upper end of this Weber number range the distortion of the drop can change the Nusselt number by about −20%. Since much greater distortion is observed at higher N_{We}, extension of the range of the above development is highly desirable.

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NOTATION

A_s = area (L^2)
D = equivalent diameter (L)
$\mathfrak{D}_{AC}$ = diffusivity in continuous phase (L^2t^{-1})
h_x, h_y, h_z = scale factors in x, y and z directions (L)
N_{Nu} = Nusselt number in continuous phase
N_{We} = Weber number
$Q_n(u)$ = defined by Equation (4)
r = radial distance (L)
R = radius of equivalent sphere (L)
v_∞ = reference velocity or velocity of spherical drop (Lt^{-1})
v_r, v_θ = velocity in r and θ directions (Lt^{-1})
v_t, v_{to} = tangential velocity and tangential velocity at the surface (Lt^{-1})
γ = ratio of the density of the interior fluid to that of the exterior
κ = ratio of viscosity of continuous phase to that of the dispersed phase
λ = parameter defined by Equation (2)
ψ = stream function (dimensionless)

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